Instructions: Try these problems from the textbook as practice (answers below).

Problems

5-20. Evaluate the surface integral.

5. $\iint_S (x+y+z)\,dS$, S is given by $x=u+v$, $y=u-v$, $z=1+2u+v$, $0\le u\le 2$, $0\le v\le 1$
7. $\iint_S y\,dS$, S is the helicoid with vector equation $\mathbf{r}(u,v)=\langle u\cos v, u\sin v, v\rangle$, $0\le u\le 1,\ 0\le v\le \pi/2$
13. $\iint_S z^2\,dS$, S is the part of $x=y^2+z^2$ given by $0\le x\le 1$
15. $\iint_S x\,dS$, S is the surface $y=x^2+4z,\ 0\le x\le 1,\ 0\le z\le 1$
17. $\iint_S (x^2z+y^2z)\,dS$, S is the hemisphere $x^2+y^2+z^2=4,\ 0\ge z$
19. $\iint_S xz\,dS$, S is the boundary of the region enclosed by the cylinder $y^2+z^2=9$ abd the plaes $x=0$ and $x+y=5$

21-32. Evaluate the surface integral $\iint_S \mathbf{F}\cdot d\mathbf{S}$ for the given vector field $\mathbf{F}$ and the oriented surface S. In other words, find the flux of $\mathbf{F}$ across S. For closed surfaces, use the positive (outward) orientation.

23. $\mathbf{F}(x,y,z)=\langle xy, yz, zx\rangle$, S is the part of the paraboloid $z=4-x^2-y^2$ that lies above the square $0\le x\le 1, 0\le y\le 1$ and has upward orientation
25. $\mathbf{F}(x,y,z)=\langle x,y,z^2\rangle$, S is the sphere with radius 1 and center the origin
27. $\mathbf{F}(x,y,z)=\langle 0,y,-z\rangle$, S consists of the paraboloid $y=x^2+z^2,\ 0\le y\le 1$ and the disk $x^2+z^2\le 1,\ y=1$
29. $\mathbf{F}(x,y,z)=\langle x,2y,3x\rangle$, S is the cube with vertices $(\pm 1,\pm 1,\pm 1)$
31. $\mathbf{F}(x,y,z)=\langle x^2,y^2,z^2\rangle$, S is the boundary of the solid half-cylinder $0\le z\le \sqrt{1-y^2},\ 0\le x\le 2$

Answers

5. $11\sqrt{14}$
7. $\frac{2}{3}(2\sqrt{2}-1)$
13. $(\pi/120)(25\sqrt{5}+1)$
15. $\frac{7\sqrt{21}}{4} - \frac{17\sqrt{17}}{12}$
17. 16π
19. 0
23. $\frac{713}{180}$
25. $\frac{8\pi}{3}$
27. 0
29. 48
31. $2\pi + \frac{8}{3}$